GCSE Maths – Algebra

Common Sequences

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of common sequences questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Identify the following type of sequence: 10, 15, 21, 28, 36, ...

Step 1: Find the difference between each of the terms, by subtracting the next term from the previous term. This is known as finding the first difference.

10, 15, 21, 28, 36,…

$+5 \quad +6 \quad +7 \quad +8$

Step 2: Analyse the pattern in the first difference and decide if you need to find the second difference.

The first differences are $+5, +6, +7, +8$*. This is indicative of a triangular sequence, where the previous term increases by one more than the term before that did.*

There is no need to find the second difference, as this is a common sequence.

The sequence is triangular.

Guided Example

Identify the following type of sequence: 1, 5, 13, 25, 41, ...

Step 1: Find the first difference between the terms, by subtracting the smaller from the greater term.

1, 5, 13, 25, 41

+4 +8 +12 +16

Step 2: Analyse the pattern in the first difference to see if the sequence is a common one.

The sequence cannot be identified based on the first difference.

Step 3: If you are unable to identify the sequence from the first difference, calculate the second difference.

4, 8, 12, 16

+4 +4 +4

Step 4: Conclude the type of sequence present. Look back at the 'Common sequences' Revision Notes if you need a reminder of how to identify the types of sequences.

This is a quadratic sequence because the sequence of differences between the terms changes by the same amount each time.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Identify the following types of sequences:

a) 2, 3, 5, 8, 13, 21, ...

$2 \xrightarrow{+1} 3 \xrightarrow{+2} 5 \xrightarrow{+3} 8 \xrightarrow{+5} 13 \xrightarrow{+8} 21$

This is the Fibonacci sequence

The term 5, 8, 13 and 21 are found by adding the previous 2 terms together.

b) 2, 6, 10, 14, 18, …

$2 \xrightarrow{+4} 6 \xrightarrow{+4} 10 \xrightarrow{+4} 14 \xrightarrow{+4} 18$

This is an arithmetic sequence as it has a common difference of +4.

c) 1, 4, 9, 16, 25, …

$1 \uparrow (1)^2$, $4 \uparrow (2)^2$, $9 \uparrow (3)^2$, $16 \uparrow (4)^2$, $25 \uparrow (5)^2$

This is a square number sequence because it has an n^{th} term of n^2.

d) 8, 27, 64, 125, …

$8 \uparrow (2)^3$, $27 \uparrow (3)^3$, $64 \uparrow (4)^3$, $125 \uparrow (5)^3$

This is a cube number sequence because it has an n^{th} term of n^3.

e) 15, 17, 19, 21, 23, …

$15 \xrightarrow{+2} 17 \xrightarrow{+2} 19 \xrightarrow{+2} 21 \xrightarrow{+2} 23$

This is an arithmetic sequence as it has a common difference of +2

f) 3, 11, 25, 45, 71, ...

$3 \rightarrow 11 \rightarrow 25 \rightarrow 45 \rightarrow 71$

$8 \xrightarrow{+6} 14 \xrightarrow{+6} 20 \xrightarrow{+6} 26$

This is a quadratic sequence because the sequence of differences changes by the same amount each time.

Section B

Worked Example

What are the next two terms in this sequence? 21, 34, 55, 89, 144, ...

Step 1: Identify the sequence by calculating the first difference.

$$\mathbf{\textit{21,}} \quad \mathbf{\textit{34,}} \quad \mathbf{\textit{55,}} \quad \mathbf{\textit{89,}} \quad \mathbf{\textit{144, ...}}$$
$$+13 \quad +21 \quad +34 \quad +55$$

The sequence increases by the previous term, and there is no common second difference. Hence, we can deduce that this is a ***Fibonacci sequence.***

Step 2: Find the next term by adding the previous two terms. This is the Fibonacci sequence rule.

$$89 + 144 = 233$$

The 6th term is **233.**

Step 3: Find the seventh term by adding the fifth and sixth terms together.

$$144 + 233 = 377$$

The 7th term is **377.**

Guided Example

What are the next two terms in this sequence? 3, 9, 27, 81, 243, ...

Step 1: Try to identify the sequence by calculating the first difference.

3 9 27 81 243
+6 +18 +54 +162

Step 2: As there is no common first difference, look at the second difference and other patterns in the sequence to conclude the type of sequence present.

Consider, is there a common second difference?
How else do each of the terms relate to each?

3 9 27 81 243
×3 ×3 ×3 ×3

The sequence is geometric sequence since they all have a common multiplier.

Step 3: Use the common multiplier to find the sixth and seventh terms.

6th term : 243 × 3 = 729

7th term : 729 × 3 = 2187

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. Continue the following sequences by finding the next two terms:

a) 8, 27, 64, 125…

8 ← 2^3, 27 ← 3^3, 64 ← 4^3, 125 ← 5^3

continue to calculate next 2 terms →

5th term : $6^3 = 216$
6th term : $7^3 = 343$

The next two terms are **216, 343**

b) 5, 12, 25, 44…

5, 12, 25, 44
+7, +13, +19
+6, +6

The next difference increases by +6.
5th term = 44 + (19 + 6) = 69
6th term = 69 + (25 + 6) = 100
difference increases by +6

The next 2 terms are **69, 100**

c) 1, 6, 36, 216…

1, 6, 36, 216
×6, ×6, ×6

They have a common multiplier of 6.
5th term : 216 × 6 = 1296
6th term : 1296 × 6 = 7776
The next 2 terms are **1296, 7776**

d) 21, 26, 31, 36…

21, 26, 31, 36
+5, +5, +5

Each subsequent term increases by +5
5th term : 36 + 5 = 41
6th term : 41 + 5 = 46
The next 2 terms are **41, 46**

e) 28, 36, 45, 55…

28, 36, 45, 55
+8, +9, +10

Triangular sequence : difference increases by +1
5th term : 55 + (10 + 1) = 66 previous difference is 10
6th term : 66 + (11 + 1) = 78
The next 2 terms are **66, 78**

f) 3, 5, 8, 12, 17…

3, 5, 8, 12, 17
+2, +3, +4, +5

Triangular sequence : difference increases by +1
6th term : 17 + (5 + 1) = 23 previous difference is 5
7th term : 23 + (6 + 1) = 30
The next 2 terms are **23, 30**